J. E. Galé and T. J. Ransford, On the growth of analytic semigroups along vertical lines, *Studia Math.*, 138 (2000), 165–177.

Abstract

We construct two Banach algebras, one which contains analytic semigroups $(a^z)_{\Re z>0}$ such that $||a^{1+iy}|| \to \infty$ arbitrarily slowly as $|y| \to \infty$, the other which contains ones such that $||a^{1+iy}|| \to \infty$ arbitrarily fast.