Y. Langlois and T. J. Ransford, Counterexample to a conjecture of Elsner on the spectral variation of matrices, *Linear Algebra Appl.*, 349 (2002), 193–195.

Abstract

In 1985, Elsner proved that the Hausdorff distance Δ between the spectra of two $n\times n$ matrices A and B satisfies

$$\Delta(\sigma(A), \sigma(B))^{n} \le (\|A\| + \|B\|)^{n-1} \|A - B\|,$$

where $\|\cdot\|$ denotes the operator norm with respect to the Euclidean norm on \mathbb{C}^n . He further conjectured that the same inequality holds for all operator norms. We disprove this conjecture, and also the weaker conjecture where $(\|A\| + \|B\|)$ is replaced by $2 \max(\|A\|, \|B\|)$.