O. El-Fallah and T. J. Ransford, Extremal growth of powers of operators satisfying resolvent conditions of Kreiss–Ritt type, J. Funct. Anal., 196 (2002), 135–154.

Abstract

Let E be a compact subset of the unit circle. We determine the extremal rate of growth of $(||T^n||)_{n\geq 1}$ for Banach-space operators T satisfying the resolvent condition

$$\|(T - \lambda I)^{-1}\| \le \frac{\text{const.}}{\text{dist}(\lambda, E)} \qquad (|\lambda| > 1).$$

This includes as extreme cases the Kreiss condition $E = \mathbf{T}$ and the Ritt condition $E = \{1\}$. For intermediate sets E, the cardinality, the measure and the Hausdorff dimension of E all play a rôle in determining the growth of $||T^n||$.

As a by-product, we also obtain lower bounds for the Taylor coefficients of functions f holomorphic on the unit disk and satisfying

$$|f(z)| \ge \frac{1}{\operatorname{dist}(z, E)}$$
 $(|z| < 1).$