V. Havin and J. Mashreghi, Admissible majorants for model subspaces of $H^2(\mathbf{R})$, Part I: slow winding of the generating inner function, *Can. J. Math.*, 55 (2003), 1231–1263.

Abstract

A model subspace K_{Θ} of the Hardy space $H^2 = H^2(\mathbf{C}_+)$ for the upper half plane \mathbf{C}_+ is $H^2(\mathbf{C}_+) \ominus \Theta H^2(\mathbf{C}_+)$ where Θ is an inner function in \mathbf{C}_+ . A function $\omega : \mathbf{R} \mapsto [0, \infty)$ is called an admissible majorant for K_{Θ} if there exists an $f \in K_{\Theta}, f \neq 0, |f(x)| \leq \omega(x)$ almost everywhere on \mathbf{R} . For some (mainly meromorphic) Θ 's some parts of Adm Θ (the set of all admissible majorants for K_{Θ}) are explicitly described. These descriptions depend on the rate of growth of arg Θ along \mathbf{R} . This paper is about slowly growing arguments (slower than x). Our results exhibit the dependence of Adm B on the geometry of the zeros of the Blaschke product B. A complete description of Adm B is obtained for B's with purely imaginary ("vertical") zeros. We show that in this case a unique minimal admissible majorant exists.