A. Blanco, M. Boumazgour and T. J. Ransford, On the norm of elementary operators, J. London Math. Soc. 70 (2004), 479–498.

Abstract

We consider the norm problem for elementary operators of the form $U_{a,b} : \mathcal{A} \to \mathcal{A}, x \mapsto axb+bxa \ (a, b \in \mathcal{A})$ in the special case when \mathcal{A} is a subalgebra of the algebra of bounded operators on a Banach space. In particular, we establish the lower estimate $||U_{a,b}|| \geq ||a|| ||b||$ when the Banach space is a Hilbert space and the algebra \mathcal{A} is the algebra of all bounded linear operators.