Javad Mashreghi and Thomas Ransford, Using entire functions to analyse power growth, *Proceedings of Banach Algebras and Their Applications*, (Eds. T. Lau, V. Runde), Contemporary Math. 263, 235–240, American Mathematical Society, Providence RI, 2004.

## Abstract

We illustrate a technique from the theory of entire functions by proving the following variant of a result of Allan about power growth in Banach algebras. Let a be an element of a unital Banach algebra, let  $m \in \mathbb{Z}^+$  and let  $\alpha \in (0, 1)$ . Then

$$\left\|a^m \left((1+a)^n - (1-a)^n\right)\right\| = O(e^{\epsilon n^\alpha}) \qquad (n \to \infty)$$

for all  $\epsilon > 0$  if and only if  $\lim_{n \to \infty} n^{1/\alpha - 1} ||a^n||^{1/n} = 0$ .