C. Costara and T. J. Ransford, **On local irreducibility of the spectrum**, *Proc. Amer. Math. Soc.*, 135 (2007), 2779-i-2784.

## Abstract

Let  $\mathcal{M}_n$  be the algebra of  $n \times n$  complex matrices, and for  $x \in \mathcal{M}_n$  denote by  $\sigma(x)$ and  $\rho(x)$  the spectrum and spectral radius of x respectively. Let D be a domain in  $\mathcal{M}_n$  containing 0, and let  $F : D \to \mathcal{M}_n$  be a holomorphic map. We prove: (1) if  $\sigma(F(x)) \cap \sigma(x) \neq \emptyset$  for  $x \in D$ , then  $\sigma(F(x)) = \sigma(x)$  for  $x \in D$ ; (2) if  $\rho(F(x)) = \rho(x)$  for  $x \in D$ , then again  $\sigma(F(x)) = \sigma(x)$  for  $x \in D$ . Both results are special cases of theorems expressing the irreducibility of the spectrum  $\sigma(x)$  near x = 0.