Conférence Prix Ribenboim

Maksym Radziwill (McGill)

Recent progress in multiplicative number theory

Multiplicative number theory aims to understand the ways in which integers factorize, and the distribution of integers with special multiplicative properties (such as primes). It is a central area of analytic number theory with various connections L-functions, harmonic analysis, combinatorics, probability, ... At the core of the subject lie difficult questions such as the Riemann Hypothesis, and they set a benchmark for its accomplishments.

An outstanding question in this field is to understand the multiplicative properties of integers linked by additive conditions, for instance n and n + 1. A central conjecture making this question precise is the Elliott-Chowla conjecture on correlations of multiplicative functions evaluated at consecutive integers. Until recently this conjecture appeared completely out of reach and was thought to be at least as difficult as showing the existence of infinitely many twin primes. These are also the kind of questions that lie beyond the capability of the Riemann Hypothesis.

However recently the landscape of analytic number theory has been evolving and several orthodoxies have been shattered, and we no longer necessarily have the illusion of a clean understanding as we did before. I will explain the progress that was accomplished and why conjectures such as the Elliott-Chowla conjecture might be in fact only a few years away from a complete resolution.

Conférence grand public

Jean-Marie De Koninck (Laval)

La vie secrète des mathématiques

Suite à un test positif fiable dans 98% des cas, votre médecin vous annonce que vous souffrez d'une maladie grave; devriez-vous être inquiet? Comment faire sauter les bouchons de circulation en utilisant les maths? Voulez-vous comprendre le processus de la survente des billets d'avion? En cette période fébrile de Coupe du monde 2018, pourquoi est-il important de marquer le premier but au soccer? À quel moment devrait-on retirer le gardien de but au hockey? Voilà quelques-unes des questions qui illustrent l'importance des mathématiques dans le fonctionnement de notre société et que Jean-Marie De Koninck aborde dans sa conférence « La vie secrète des mathématiques ».

Henri Darmon (McGill)

The j function, the golden ratio, and p-adic meromorphic cocycles

Last year Jan Vonk and I introduced the notion of "rigid meromorphic cocycle" with the goal of extending the theory of complex multiplication and of singular moduli to the setting of real quadratic fields. I will describe these mathematical objects with special emphasis on one of its "simplest" instances, a 3-adic meromorphic cocycle whose zeroes and poles are concentrated on real quadratic irrationalities of the form $(a\varphi + b)/(c\varphi + d)$, where φ is the golden ratio and a, b, c, d are elements of $\mathbb{Z}[1/3]$ with $ad - bc = 1$.

Andrew Granville (Montréal)

Recent developments on the "pretentious approach" to analytic number theory

In this talk we will discuss some of the latest developments on our understanding of the mean value of multiplicative functions in short intervals (due to various subsets of Harper, Matomaki, Radziwill, Soundararajan and the speaker), as well as in arithmetic progresssions (subsets of Drappeau, Shao and the speaker). We will focus on identifying limitations, whether to the techniques used or to the actual range on which we expect the purported estimates to be true.

Joseph Oesterlé (Paris VI)

Multiple zeta values and multiple Apéry-like sums (after P. Akhilesh)

In this talk, we shall introduce the notion of multiple Apéry-like sums and show that every multiple zeta value can be expressed as a Z-linear combination of them. There is even a canonical way to do so. This allows to put in a unified theoretical context several identities scattered in the literature, as well as to discover a large number of surprisingly new ones.

Ken Ono (Emory)

Polya's Program for the Riemann Hypothesis and Related Problems

In 1927 Polya proved that the Riemann Hypothesis is equivalent to the hyperbolicity of Jensen polynomials for Riemann's Xi-function. This hyperbolicity has only been proved for degrees d=1, 2, 3. We prove the hyperbolicity of all (but possibly finitely many) the Jensen polynomials of every degree d. We obtain a general theorem which models such polynomials by Hermite polynomials, which in the case of the zeta-function can be thought of as a "degree aspect" GUE distribution. The general theorem also allows us to prove a conjecture of Chen, Jia, and Wang on the partition function. This is joint work with Michael Griffin, Larry Rolen, and Don Zagier.

Bjorn Poonen (MIT)

Heuristics for the arithmetic of elliptic curves

I will discuss a probabilistic model for the arithmetic of elliptic curves, developed in a series of papers with Manjul Bhargava, Daniel M. Kane, Hendrik Lenstra, Jennifer Park, Eric Rains, John Voight, and Melanie Matchett Wood.

Damien Roy (Ottawa)

Parametric geometry of numbers

Rational approximation to an n-tuple of real numbers can be analyzed from different points of view, leading to a variety of so-called exponents of Diophantine approximation. These exponents are not independent of one another and a basic problem is to determine their joint spectrum, namely the set of all values that they take as the n-tuple varies.

Parametric geometry of numbers, initiated by Schmidt and Summerer in 2009-13, is a powerful tool for this purpose. It is motivated by the observation that the usual exponents of Diophantine approximation to a point can be computed in terms of the successive minima of a one parameter family of convex bodies attached to that point. The main result of the theory characterizes, up to bounded difference, all functions that arise as the n-tuple of logarithms of the successive minima of such families. This is done in terms of a simple class of functions called n-systems. As a consequence, the problem of describing the joint spectrum of a family of exponents of Diophantine approximation is reduced to combinatorial analysis. In this talk, we present the theory and survey some of its recent applications.

Dinesh Thakur (Rochester)

Zeta, Multizeta and emerging related structures in function field arithmetic

Will survey recent developments in quest for understanding zeros and nature of special values through related abelian and non-abelian structures, such as extensions of Anderson's t-motives, big Galois representations and algebraic groups.

Jared Weinstein (Boston University)

New developments in p-adic geometry

This will be a gentle exposition of some recent trends in arithmetic geometry over the p-adic numbers. The general idea is to replace p with a variable, so that arithmetic looks more like geometry. As an example, we discuss a geometric object (a "diamond") whose geometric fundamental group is isomorphic to the Galois group of the p-adic numbers.

Melanie Wood (Wisconsin-Madison)

Effective Chebotarev density theorems for families of number fields without GRH

We discuss a new effective Chebotarev density theorem, not conditional on GRH, that improves the previously known unconditional error term and allows primes to be taken quite small; this theorem holds for the Galois closures of "almost all" number fields that lie in an appropriate family of field extensions. We discuss applications to bounds on l-torsion in the class groups and small generators for number fields. This talk is on joint work with Lillian Pierce and Caroline Turnage-Butterbaugh.