Maine-Québec
Number Theory Conference

University of Maine
October 2nd and 3rd, 2021

Organizers

Jack Buttcane
Andrew Knightly

Saturday			
	Virtual Room A	Virtual Room B	Virtual Room C
9:00-9:10	Welcome		
9:10-10:00	**Valentin Blomer** (Universität Bonn) *Density theorems for* $GL(n)$		
10:00-10:10	Break		
10:10-10:30	**Robert Hough** (SUNY Stony Brook) *Subconvexity of the Shintani zeta function*	**Antonio Lei** (Université Laval) *Anticyclotomic Selmer groups for CM elliptic curves at inert primes*	
10:30-10:40	Break		
10:40-11:00	**Eun Hye Lee** (Stony Brook University) *Multiple Dirichlet series associated to binary cubic forms*	**Wanlin Li** (CRM) *Abelian varieties of prescribed order over finite fields*	
11:00-11:30	Coffee		
11:30-11:50	**David Lowry-Duda** (ICERM) *Zeros of Half Integral Weight Dirichlet Series*	**Jaitra Chattopadhyay** (Indian I.T. Guwahati) *On Some Families of Non-Pólya Bi-quadratic Fields*	
11:50-12:00	Break		
12:00-12:20	**Kunjakanan Nath** (Université de Montréal) *Distribution of primes with a missing digit in arithmetic progressions*	**Katharina Müller** (Université Laval) *Iwasawa* μ *invariants of fine Selmer groups*	
12:20-2:00	Lunch		
2:00-2:20	**Saurabh Singh** (Indian Institute of Technology) *Delta symbol and its applications*	**Subramani Muthukrishnan** (IIIT D&M, Chennai) *Euclidean Quartic number fields*	
2:20-2:30	Break		
2:30-2:50	**Joshua Zelinsky** (Hopkins School) *Mertens' Theorem and perfect numbers*	**Anwesh Ray** (University of British Columbia) *Constructing Galois representations ramified at one prime*	
2:50-3:00	Break		
3:00-3:20	**John Voight** (Dartmouth College) *Stickelberger's discriminant theorem for algebras*	**Debanjana Kundu** (UBC Vancouver/ PIMS) *Arithmetic Statistics and Iwasawa Theory*	
3:20-3:50	Coffee		
3:50-4:05 (students)	**Krystian Gajdzica** (Jagiellonian University) *Arithmetic properties of the restricted partition function* $p_{\mathcal{A}}(n,k)$	**Pavel Čoupek** Video (Purdue University) *Ramification bounds for mod* p *étale cohomology via prismatic cohomology*	**Tariq Osman** (Queen's University) *A New Bound for a Particular Family of Quadratic Weyl Sums*
4:05-4:15	Break		
4:15-4:30 (students)	**Nihar Gargava** (EPFL) *Lattice packings through division algebras*	**Elad Zelingher** (Yale University) *On regularization of integrals of matrix coefficients associated with spherical models*	**Stelios Sachpazis** (Université de Montréal) *On multiplicative functions with small averages*
4:30-4:40	Break		
4:40-4:55 (students)	**Peter Zenz** (McGill University) *Quantum Variance for holomorphic Hecke cusp forms on the vertical geodesic*	**Siva Nair** (Université de Montréal) *A Family of Polynomials with Mahler Measure* $\frac{28}{5\pi^2}\zeta(3)$	**Luochen Zhao** (Johns Hopkins University) *Sum expressions for Kubota-Leopoldt p-adic L-functions*
4:55-5:05	Break		
5:05-5:20 (students)	**Alexandre de Faveri** (Caltech) *Simple zeros of GL(2) L-functions*	**Isabella Negrini** (McGill University) *A Shimura-Shintani correspondence for rigid analytic cocycles*	**Chung-Hang Kwan** Video (Columbia University) *Moment formulae for automorphic L-functions of higher-rank groups*

Sunday			
	Virtual Room A	Virtual Room B	Virtual Room C
9:00-9:20	**Min Lee** (University of Bristol) *Non-vanishing of symmetric cube L-functions*		
9:20-9:30	Break		
9:30-9:50	**Jonathan Bayless** (Univ. of Maine at Augusta) *Higher Mertens constants for almost primes*	**Abbas Maarefparvar** Video (Inst. for Research in Fund. Sciences) *Polya-Ostrowski groups in algebraic number fields*	
9:50-10:00	Break		
10:00-10:20	**Yeongseong Jo** (The University of Maine) *The weak test vector problems and local periods*	**Matilde Lalin** (Université de Montréal) *Sums of certain arithmetic functions over $\mathbb{F}_q[T]$ and symplectic distributions*	
10:20-10:50	Coffee		
10:50-11:10	**Edmund Karasiewicz** (University of Utah) *The twisted Satake transform and the Casselman-Shalika formula*	**Jonathan Love** (McGill University (CRM-ISM)) *Products of Pythagorean slopes*	
11:10-11:20	Break		
11:20-11:40	**Kim Klinger-Logan** (Kansas State University) *Graviton Scattering and Differential Equations in Automorphic Forms*	**Sudhir Pujahari** (University of Warsaw) *Distribution of moments of traces of Frobenius in arithmetic progressions*	
11:40-11:50	Break		
11:50-12:10	**Joshua Males** (University of Manitoba) *Mock modular forms and Eichler-Selberg relations*	**Manami Roy** (Fordham University) *Local data for rational elliptic curves with non-trivial torsion*	
12:10-12:20	Break		
12:20-12:40	**Eric Stubley** (McGill University) *Classical forms of low weight in p-adic families of ordinary modular forms*	**Andrew Schultz** (Wellesley College) *Realizing module structures of square power classes over biquadratic extensions*	
12:40-2:00	Lunch		
2:00-2:50	**Theresa Anderson** (Purdue University) *Two meetings of number theory and analysis*		
2:50-3:00	Break		
3:00-3:15 (students)	**Hari Iyer** (Harvard College) *One-level density of a family of L-functions over function fields*	**Subham Roy** (Université de Montréal) *Mahler measure of a family of polynomials for arbitrary tori*	**Félix Baril Boudreau** (University of Western Ontario) *Computing an L-function modulo a prime*
3:15-3:25	Break		
3:25-3:40 (students)	**Simran Khunger** (Carnegie Mellon University) *Extending support in calculating the n^{th} moment of the 1-level density of low lying zeroes of orthogonal families*	**George Hauser** (Rutgers University) *Eisenstein Series on Higher Covers of SL(2,R)*	**Cédric Dion** (Université Laval) *Functional equation for chromatic Selmer groups*
3:40-3:50	Break		
3:50-4:05 (students)	**Doyon Kim** (Rutgers University) *Automorphic distributions and Zeros of L-functions*	**Yusheng Lei** (Boston College) *Theta liftings on double covers of orthogonal groups*	**Anthony Doyon** (Université Laval) *Congruences between additive elliptic curves and Ramanujan's tau function*
4:05-4:15	Break		
4:15-4:30 (students)	**Grant Molnar** (Dartmouth College) *The LCM product and Gronwall's theorem*	**Zhining Wei** (Ohio State University) *Linear Relations of Siegel Poincaré Series and Non-vanishing of the Central Values of Spinor L-functions*	**Tyler Genao** (University of Georgia) *Typically bounding torsion on elliptic curves: $j(E) \in F$ and beyond*

Abstracts

Theresa Anderson, Purdue University

Two meetings of number theory and analysis

In many recent works, number theory and analysis go beyond working side by side and team up in an interconnected back and forth interplay to become a powerful force. Here I describe two distinct meetings of the pair, which result in sharp counts for equilateral triangles in Euclidean space and statistics for how often a random polynomial has Galois group not isomorphic to the full S_n.

Félix Baril Boudreau, University of Western Ontario (student)

Computing an L-function modulo a prime

Let E be an elliptic curve with non-constant j-invariant over a function field K with constant field of size an odd prime power q. Its L-function $L(T, E/K)$ belongs to $1 + T\mathbb{Z}[T]$. Inspired by the algorithms of Schoof and Pila for computing zeta functions of curves over finite fields, we propose an approach to compute $L(T, E/K)$. The idea is to compute, for sufficiently many primes ℓ invertible in K, the reduction $L(T, E/K) \bmod \ell$. The L-function is then recovered via the Chinese remainder theorem. When $E(K)$ has a subgroup of order $N \geq 2$ coprime with q, Chris Hall showed how to explicitly calculate $L(T, E/K) \bmod N$. We present novel theorems going beyond Hall's.

Jonathan Bayless, University of Maine at Augusta

Higher Mertens constants for almost primes

For $k \geq 1$, a k-almost prime is a positive integer with exactly k prime factors, counted with multiplicity. In this article we give elementary proofs of precise asymptotics for the reciprocal sum of k-almost primes. Our results match the strength of those of classical analytic methods. We also study the limiting behavior of the constants appearing in these estimates, which may be viewed as higher analogues of the Mertens constant $\beta = 0.2614\ldots$ This is joint work with Paul Kinlaw and Jared Lichtman.

Valentin Blomer, Universität Bonn

Density theorems for $GL(n)$

The generalized Ramanujan conjecture predicts that all cuspidal automorphic representations for $GL(n)$ are tempered. A density theorem is a quantitative version of the statement that non-tempered representations become rarer the further their Langlands parameters at a given place are away from the unitary axis. In many cases this is a good substitute for the

Ramanujan conjecture, in the same way as the Bombieri-Vinogradov theorem can be used a substitute for the Riemann hypothesis for Dirichlet L-functions. In this talk I show how a relative trace formula together with new bounds for $GL(n)$ Kloosterman sums can obtain strong density theorems, and I present some applications.

Jaitra Chattopadhyay, Indian Institute of Technology Guwahati

On Some Families of Non-Pólya Bi-quadratic Fields

An algebraic number field K is called a Pólya field if the ring of integer-valued polynomials has a regular basis. This behavior is governed by the Pólya group $Po(K)$, a particular subgroup of the ideal class group Cl_K of K. The study of Pólya groups for various number fields is of considerable interest in recent times. In this talk, we shall compute the Pólya groups of three possibly infinite families of bi-quadratic fields, two of which are known to have non-principal Euclidean ideal class under mild conditions. This is a joint work with Anupam Saikia.

Pavel Čoupek, Purdue University (student)

Ramification bounds for mod p étale cohomology via prismatic cohomology

Given a smooth proper formal scheme X over $\mathcal{O}_K$ where K is a p–adic field and p is an odd prime, we give an upper bound for ramification of the $\mathbb{F}_p$–representations $H^i_{\acute{e}t}(X_{\mathbb{C}_K}, \mathbb{Z}/p\mathbb{Z})$ in terms of p, i, and e, the absolute ramification index of K, without any restriction on the size of i and e. In order to achieve this, a crucial input is the recently developed prismatic cohomology in its Breuil–Kisin and A_{inf}–instances, $H^i_{\Delta}(X/\mathfrak{S})$ and $H^i_{\Delta}(X_{A_{\mathrm{inf}}}/A_{\mathrm{inf}})$, resp., and a series of conditions $(\mathrm{Cr}_s)_{s\geq 0}$ that control the Galois action on the elements of the Breuil–Kisin cohomology groups inside the A_{inf}–cohomology groups.

VIDEO OF LECTURE

Alexandre de Faveri, Caltech (student)

Simple zeros of GL(2) L-functions

Let $f \in S_k(\Gamma_1(N))$ be a primitive holomorphic form of arbitrary weight k and level N. We show that the completed L-function of f has $\Omega(T^{\delta})$ simple zeros with imaginary part in $[-T, T]$, for any $\delta < \frac{2}{27}$. This is the first power bound in this problem for f of non-trivial level, where previously the best results were $\Omega(\log\log\log T)$ for N odd, due to Booker, Milinovich, and Ng, and infinitely many simple zeros for N even, due to Booker. In addition, for f of trivial level ($N = 1$), we also improve an old result of Conrey and Ghosh on the number of simple zeros.

Cédric Dion, Université Laval (student)
Functional equation for chromatic Selmer groups
Let p be a fixed odd prime and let E be an elliptic curve over the rational numbers. Suppose that E has good ordinary reduction at p. It is well known that the p-adic L-function of E over the cyclotomic extension satisfies a functional equation. It also follows from Mazur's work that the p-primary Selmer group of E over the same extension satisfies a so-called algebraic functional equation. In this talk, we investigate the setting where E has good supersingular reduction at p and the Selmer groups considered are Sprung's chromatic Selmer groups. We show that these Selmer groups also satisfy a functional equation over the cyclotomic extension.

Anthony Doyon, Université Laval (student)
Congruences between additive elliptic curves and Ramanujan's tau function
Let E denote an elliptic curve defined over $\mathbb{Q}$. By modularity, one can associate a modular form f_E to E, say $f_E(z) = \sum_{n\geq 1} a_n(E)q^n$ where $q = e^{2\pi iz}$. Let Δ be the unique normalized cuspidal modular form of weight 12 and level 1 given by the following q-expansion

$$\Delta(z) = q\prod_{n\geq 1}(1-q^n)^{24} = \sum_{n\geq 1}\tau(n)q^n$$

where $\tau : \mathbb{N} \to \mathbb{Z}$ is known as Ramanujan's tau function. Let p be a prime. This talk focuses on the following question: For which elliptic curves E does $a_n(E) \equiv \tau(n) \pmod{p}$ holds for all $n \geq 1$?
For $p = 2$ and 3, we present families of elliptic curves satisfying this condition. We also study the link between p-adic Mazur-Tate elements attached to Δ and those attached to certain elliptic curves E having additive reduction at p. Under certain hypotheses, we can prove an equality of p-adic Iwasawa μ and λ invariants of Mazur-Tate elements.

Krystian Gajdzica, Jagiellonian University (student)
Arithmetic properties of the restricted partition function $p_{\mathcal{A}}(n,k)$
Let $\mathcal{A} = (a_n)_{n\in\mathbb{N}_+}$ be a sequence of positive integers. The function $p_{\mathcal{A}}(n,k)$ counts the number of multi-color partitions of n into parts in $\{a_1,\ldots,a_k\}$. We present several arithmetic properties of the sequence $(p_{\mathcal{A}}(n,k) \pmod{m})_{n\in\mathbb{N}}$ for an arbitrary fixed integer $m \geqslant 2$, and apply them to the special cases of $\mathcal{A}$. In particular, for a fixed parameter k, we investigate both lower and upper bounds for the odd density of $p_{\mathcal{A}}(n,k)$. Moreover, we perform some new results related to restricted m-ary partitions and state a few open questions at the end of the talk.

Nihar Gargava, EPFL, Switzerland (student)
Lattice packings through division algebras
We will show the existence of lattice packings in a sparse family of dimensions. This construction will be a generalization of Venkatesh's lattice packing result of 2013. In our construction, we replace the appearance of the cyclotomic number field with a division algebra over the rationals. This improves the best known lower bounds on lattice packing problem in many dimensions. The talk will cover previously known bounds, an overview of the new bounds and a live numerical simulation of Siegel's mean value theorem.

Tyler Genao, University of Georgia (student)
Typically bounding torsion on elliptic curves: $j(E) \in F$ and beyond
In 1996, Merel proved the "strong uniform boundedness conjecture" for torsion subgroups of elliptic curves over number fields of a fixed degree, suggested by previous work of Mazur, Kamienny, Kenku and Momose. Unfortunately, explicit upper bounds on torsion subgroups over degree $d > 3$ number fields are greater than exponential in d. Despite this, special families of elliptic curves, such as the family of elliptic curves with complex multiplication, have torsion subgroups which satisfy a "typical order": there exists a bound on torsion subgroups of this family which works over *all* degrees, provided one ignores those torsion subgroups whose number field degrees lie in a subset of $\mathbb{Z}^+$ of arbitrarily small upper density. Such families of elliptic curves are said to be *typically bounded in torsion.* In this talk, we will explore some of the main ideas involved in proving that the family $\mathcal{E}_F$ of elliptic curves with j-invariant lying in a fixed number field F is typically bounded in torsion. We will also show why this implies that a certain larger family of elliptic curves is typically bounded in torsion, and see how such a family fits into a general study of $\mathbb{Q}$-curves.

George Hauser, Rutgers University (student)
Eisenstein Series on Higher Covers of SL(2,R)
Eisenstein series on covers of SL(2) are the subject of much interest in the theory of higher theta functions, although it is necessary to extend the base field to contain roots of unity. In this talk, I will explore a construction of some Eisenstein series on high covers of SL(2,R) without extending the base field. The main result is the computation of the poles and residues of this series, which are automorphic forms on the universal cover of SL(2,R). For technical convenience we will use language of automorphic distributions.

Robert Hough, SUNY Stony Brook
Subconvexity of the Shintani zeta function
We prove a subconvex estimate for Shintani's zeta function enumerating binary cubic forms, and for its Maass form twisted version. The estimate uses methods developed by Bhargava

for enumerating orbits in representation spaces, and is expected to obtain subconvex estimates for other zeta functions of prehomogeneous vector spaces. The estimates are novel as the degree is high. Joint work with Eun Hye Lee.

Hari Iyer, Harvard College (student)
One-level density of a family of L-functions over function fields
Katz and Sarnak conjectured a correspondence between n-level density statistics of zeroes of families of L-functions and the eigenvalues of random matrix ensembles. The particular ensemble depends on the symmetry type of the family, which is a classical compact group (unitary, symplectic, or orthogonal). The latter are often studied by random matrix theory (RMT). We build upon previous work by Waxman, which has shown that L-functions associated with Hecke characters on the Gaussian integers $\mathbb{Z}[i]$ have zeroes which are modeled by the eigenvalues of symplectic matrices. We consider analogous L-functions associated with "super-even" characters in the function field setting. Though these characters have been studied from an RMT perspective as $q \to \infty$ (for $\mathbb{F}_q[t]$), we instead consider the limit where the degree K of the modulus of the Dirichlet character is large; also note that this is equivalent to the large conductor limit, since $K \log q$ is proportional to the average logarithmic conductor of the family of super-even L-functions evaluated at $s = 1/2$. We compute the limiting one-level density for this family of L-functions and we show that it matches a symplectic distribution for a class of test functions f whose Fourier transform $\hat{f}$ is compactly supported in $(-1, 1)$. We directly calculate the main term and a lower order term for the one-level density. In addition, we apply the L-functions Ratios Conjecture to compute the one-level density, and we show agreement with the unconditional result for restricted support to arbitrarily lower order $O(K^{-a})$ for all $a > 1$.

Yeongseong Jo, The University of Maine
The weak test vector problems and local periods
Let F be a non-archimedean local field of characteristic zero and π an irreducible admissible generic representation of $GL_m(F)$. By definition, the local L-function associated to π is a priori given by a finite sum of Rankin-Selberg type integrals. The purpose of this talk is to finds a pair of Whittaker functions and Schwartz-Bruhant functions so that so-called the formal L-function can be expressed as a single integral. This is referred to the weak test problem and in general the formal L-function divides the original one. Time permitting, we explain how one can extend it to archimedean local fields. The last part is joint work with Peter Humphries.

Edmund Karasiewicz, University of Utah
The twisted Satake transform and the Casselman-Shalika formula

The Fourier coefficients of automorphic forms are an important object of study due to their connection to L-functions. In the adelic framework, constructions of L-functions involving Fourier coefficients (e.g. Langland-Shahidi and Rankin-Selberg methods) naturally lead to spherical Whittaker functions on p-adic groups. Thus we would like to understand these spherical Whittaker functions to better understand L-functions. Casselman-Shalika determined a formula for the spherical Whittaker functions, and basic algebraic manipulations reveal that their formula can be more succinctly expressed in terms of characters of the Langlands dual group. We will describe a new proof of the Cassllman-Shalika formula that provides a conceptual explanation of the appearance of characters.

Simran Khunger, Carnegie Mellon University (student)
Extending support in calculating the n^{th} moment of the 1-level density of low lying zeroes of orthogonal families of L-functions
We study low-lying zeroes of L-functions and their n-level density, which relies on a smooth test function ϕ whose Fourier transform has compact support. Assuming GRH, we establish tools to compute the n^{th} centered moments of the 1-level density of low-lying zeroes of L-functions associated with weight k, prime level N cuspidal newforms as $N \to \infty$, where $\hat{\phi}$ has support in $\left(\frac{1}{n-a}, \frac{1}{n-a}\right)$ for $n \geq 2a$. The Katz-Sarnak Density conjecture claims that the n-level density of certain families of L-functions is the same as the distribution of eigenvalues of a family of orthogonal random matrices. Our work helps establish new evidence for the Katz-Sarnak Density conjecture in larger support and aids in bounding the order of vanishing of L-functions at the central point.

Doyon Kim, Rutgers University (student)
Automorphic distributions and Zeros of L-functions
The automorphic distribution attached to a holomorphic modular form $f(z)$ is defined to be the boundary values $\tau(x) = \lim_{y\to 0+} f(x+iy)$. We can define the Mellin transform of the distribution $\tau(x)$ and derive the functional equation of the L-function of $f(z)$. In this talk, I will define the automorphic distribution, present some of their properties and describe how we can prove that there are infinitely many zeros of the L-function on the critical line using automorphic distributions.

Kim Klinger-Logan, Kansas State University
Graviton Scattering and Differential Equations in Automorphic Forms
Green, Russo, and Vanhove have shown that the scattering amplitude for gravitons (hypothetical particles of gravity represented by massless string states) is closely related to automorphic forms through differential equations. Green, Miller, Russo, Vanhove, Pioline,

and K-L have used a variety of methods to solve eigenvalue problems for the invariant Laplacian on different moduli spaces to compute the coefficients of the scattering amplitude of four gravitons. We will discuss recent work with S. Miller and D. Radchenko to solve the most complicated of these differential equations $(\Delta - \lambda_s)u = E_a^2$ on $SL_2(\mathbb{Z})\backslash\mathfrak{h}$.

Debanjana Kundu, UBC Vancouver/ PIMS

Arithmetic Statistics and Iwasawa Theory

We will report on recent work (with collaborators) where we study the average behaviour of the Iwasawa invariants for the Selmer groups of elliptic curves, setting out new directions in arithmetic statistics and Iwasawa theory.

Chung-Hang Kwan, Columbia University (student)

Moment formulae for automorphic L-functions of higher-rank groups

The central values of L-functions are of great arithmetic interests. One of the ways to access their statistics is via moment calculations and moment problems of L-functions have been central to analytic number theory. There have been many spectacular advances and applications in the past decades. In this talk, we will focus on the higher-rank situation, discussing the techniques, hurdles, and some of my on-going work in this direction.

VIDEO OF LECTURE

Matilde Lalin, Université de Montréal

Sums of certain arithmetic functions over $\mathbb{F}_q[T]$ and symplectic distributions

In 2018 Keating, Rodgers, Roditty-Gershon and Rudnick established relationships of the mean-square of sums of the divisor function $d_k(f)$ over short intervals and over arithmetic progressions for the function field $\mathbb{F}_q[T]$ to certain integrals over the ensemble of unitary matrices. We consider similar problems leading to distributions over the ensemble of symplectic matrices. We also consider analogous questions involving convolutions of the von Mangoldt function. This is joint work with Vivian Kuperberg.

Eun Hye Lee, Stony Brook University

Multiple Dirichlet series associated to binary cubic forms

In this talk, I will construct a double Dirichlet series associated to a space of certain binary cubic forms and show the domain of meromorphy continues to the whole of $\mathbb{C}^2$.

Min Lee, University of Bristol

Non-vanishing of symmetric cube L-functions

The non-vanishing of L-series at the central of the critical strip has long been a subject of great interest. The main purpose of this talk is to prove that there are infinitely many Maass–Hecke cuspforms over the imaginary quadratic field of discriminant -3 such that the central values of their symmetric cube L-functions do not vanish. This is done by using a result of Ginzburg, Jiang and Rallis which shows that the non-vanishing at the central point of the critical strip of the symmetric cube L-series of any GL(2) automorphic form is equivalent to the non-vanishing of a certain triple product integral. We use spectral theory and the properties of the cubic theta function to show that the non-vanishing of this triple product occurs for infinitely many cusp forms. This is a joint work with Jeff Hoffstein and Junehyuk Jung.

Antonio Lei, Université Laval

Anticyclotomic Selmer groups for CM elliptic curves at inert primes

Let $E/\mathbb{Q}$ be a CM elliptic curve and K an imaginary quadratic field where p is inert. The goal of this talk is to shed light on the structure of the fine Selmer group of E over the anticyclotomic extension K_∞ of K, which was initially systematically studied by Coates and Sujatha in early 2000's. Burungale, Kobayashi and Ota recently proved a longstanding conjecture of Rubin on the structure of the local points on E over K_∞ from the 1980's. We will disucss how this allows us to link the fine Selmer group to the plus and minus Selmer groups of E over K_∞. This generalizes (and simplifies) a recent result of myself and Sujatha in the cyclotomic setting.

Yusheng Lei, Boston College (student)

Theta liftings on double covers of orthogonal groups

In 2003, Bump-Friedberg-Ginzburg constructed the generalized theta representations on the double cover of odd special orthogonal groups. The same authors later use such theta representations to construct the generalized theta lifting of a cuspidal automorphic representation on the double cover of an odd special orthogonal group to the double cover of an even special orthogonal group. For a fixed automorphic representation, one may ask what the smallest rank for an even special orthogonal group is so that the theta lifting to its double cover is non-zero. The case of generic representations was treated by Bump-Friedberg-Ginzburg. In this talk, I will talk about the general framework of the generalized theta liftings and some progress that predicts the first non-zero occurrence of the liftings of a fixed non-generic cuspidal automorphic representation.

Wanlin Li, CRM

Abelian varieties of prescribed order over finite fields

Given a prime power q and a positive integer n, one can ask whether every integer in the interval given by the Hasse–Weil bound is the order of some abelian variety of dimension n over F_q. In this talk, I will discuss results on this topic from joint work with Raymond van Bommel, Edgar Costa, Bjorn Poonen and Alexander Smith. Given a prime power q and $n \gg 1$, we prove that every integer in a large subinterval of the Hasse–Weil interval is the order of a geometrically simple ordinary principally polarized abelian variety of dimension n over F_q. As a consequence, we generalize a result of Howe and Kedlaya for F_2 to show that for each q, every sufficiently large positive integer is realizable as the order of an abelian variety over F_q. Our result also improves upon the best known constructions of sequences of simple abelian varieties with point counts towards the extremes of the Hasse–Weil interval. We also determine, for fixed n, the largest subinterval of the Hasse–Weil interval consisting of realizable integers, asymptotically as q grows; this gives an asymptotically optimal improvement of a 1998 theorem of DiPippo and Howe. Our methods are effective and in particular we prove that if $q \leq 5$, then every positive integer is realizable.

Jonathan Love, McGill University (CRM-ISM)

Products of Pythagorean slopes

A "Pythagorean slope" is a ratio of the lengths of the legs of a rational right triangle; by a classical parametrization, this is equivalent to being of the form $\frac{2t}{1-t^2}$ for some $t \in \mathbb{Q} \cap (0,1)$. Products of Pythagorean slopes naturally arise when studying certain configurations of points in $\mathbb{R}^n$ with pairwise rational distances. In this talk, we will classify the set of products of two Pythagorean slopes using a family of elliptic curves, and use this classification to prove that every positive rational number can be written as a product of three Pythagorean slopes (in infinitely many ways).

David Lowry-Duda, ICERM

Zeros of Half Integral Weight Dirichlet Series

Automorphic L-functions are well-studied and are believed to satisfy a number of major conjectures, such as the Riemann hypothesis. Dirichlet series associated to half-integral weight modular forms are in many ways similar to their better-studied cousins, but are not L-functions and in various ways behave much worse. In this talk, I'll describe an ongoing investigation into half-integral weight modular forms and their Dirichlet series. We'll see that these series don't satisfy a Riemann hypothesis, but are still in many ways understandable. This includes joint work with Thomas Hulse, Li-Mei Lim, and Mehmet Kiral.

Abbas Maarefparvar, Institute for Research in Fundamental Sciences (IPM)

Polya-Ostrowski groups in algebraic number fields

Pólya fields are number fields whose Bhargava generalized factorials are principal ideals (This is a modern definition, classically they are defined in terms of regular bases, or back at the time of Hilbert in terms of the action of the Galois group on the class group of a number field). As a "measure of the obstruction" for a number field to be a Pólya field, the notion of Pólya group was introduced later in 1997 by Cahen-Chabert, and a number field is Pólya iff its Pólya group is trivial. Pólya fields encompass all PID number fields, however they are more tractable than the PID ones. Whereas Gauss' class number one conjecture is still wide open for the real quadratic fields and it took a while to prove that Gauss' list of imaginary class number one fields were complete (Baker-Stark-Heegner), all Pólya quadratic fields are completely classified, essentially thanks to Hilbert's computation of ambiguous ideals as demonstrated by Pólya (more generally, Pólya groups in Galois extensions will be "controllable" throughout Galois cohomology and ramifications). In this talk, I would like to give a brief history of Pólya fields and Pólya groups along with many various examples.

VIDEO OF LECTURE

Joshua Males, University of Manitoba

Mock modular forms and Eichler-Selberg relations

Eichler-Selberg type relations for scalar-valued mock modular forms were proven by Mertens in a celebrated paper in Adv. Math (2016), thereby proving an older conjecture of Cohen. In two recent papers, the speaker and A. Mono have extended these ideas to the vector-valued world. The central idea is to use certain "higher" theta lifts and Serre duality, along with various properties of harmonic Maass forms. We will see that a wide class of vector-valued mock modular forms satisfy Eichler-Selberg relations. In a slightly different direction, we also offer a construction of locally harmonic Maass forms on grassmanians of even lattices of any indefinite signature.

Grant Molnar, Dartmouth College (student)

The LCM product and Gronwall's theorem

In this talk, we define the LCM product, due originally to R. D. von Sterneck. We relate the LCM product to Dirichlet convolution, and then prove an analogue to Grönwall's Theorem for kth LCM powers of the identity function on the natural numbers.

Katharina Müller, Université Laval

Iwasawa μ invariants of fine Selmer groups

We compare Iwasawa Invariants of fine Selmer groups of congruent abelian varieties over suitable p-adic Lie-extensions and also give a relation to ideal class groups. This is joint work with Sören Kleine.

Subramani Muthukrishnan, Indian Institute of Information Technology D&M, Chennai

Euclidean Quartic number fields

We prove that all abelian quartic number fields of class number one are Euclidean.

Siva Nair, Université de Montréal (student)

A Family of Polynomials with Mahler Measure $\frac{28}{5\pi^2}\zeta(3)$

The Mahler measure of a Laurent polynomial $P(x_1, \ldots, x_n)$ is the integral of $\log|P|$ over the unit n-torus defined by $|x_i| = 1$ for all i. Interest in this quantity arose from the fact that the Mahler measure of certain polynomials is quite remarkable and not just any random real number. If P is univariate, then the measure is given by Jensen's formula in terms of its roots, and in the multivariable case, it has been observed to yield special values of L-functions. Here, we will discuss ongoing work (with Prof. Matilde Lalín) that evaluates the Mahler measure of a family of polynomials in terms of $\zeta(3)$.

Kunjakanan Nath, Université de Montréal

Distribution of primes with a missing digit in arithmetic progressions

One of the fundamental questions in number theory is to find primes in any subset of the natural numbers. In general, it's a difficult question and leads to open problems like the twin prime conjecture, Landau's problem and many more. Recently, Maynard considered the set of natural numbers with a missing digit and showed that it contains infinitely many primes whenever the base $b \geq 10$. In fact, he has established the right order of the upper and the lower bounds when the base $b = 10$ and an asymptotic formula whenever b is large. In this talk, we will consider the distribution of primes with a missing digit in arithmetic progressions for base b large enough. In particular, we will show an analogue of the Bombieri-Vinogradov type theorems for primes with a missing digit for large base b. The proof relies on the circle method, which in turn is based on the Fourier structure of the digital set and the Fourier transform of primes over arithmetic progressions on an average. Finally, we will give its application to count the primes of the form $p = 1 + m^2 + n^2$ with a missing digit for a large odd base.

Isabella Negrini, McGill University (student)

A Shimura-Shintani correspondence for rigid analytic cocycles

In their paper *Singular moduli for real quadratic fields: a rigid analytic approach*, Darmon and Vonk introduced rigid meromorphic cocycles, i.e. elements of $H^1(\mathrm{SL}_2(\mathbb{Z}[1/p]), \mathcal{M}^\times)$ where $\mathcal{M}^\times$ is the multiplicative group of rigid meromorphic functions on the p-adic upper-half plane. Their values at RM points belong to narrow ring class fields of real quadratic fiends and behave analogously to CM values of modular functions on $\mathrm{SL}_2(\mathbb{Z})\backslash\mathcal{H}$. In this talk we will present some of our results on rigid meromorphic and analytic cocycles of weight k, as well as some of the progress we made towards developing a Shimura Shintani correspondence for rigid analytic cocycles.

Tariq Osman, Queen's University (student)

A New Bound for a Particular Family of Quadratic Weyl Sums

Consider quadratic Weyl sums of the form $S_N(x;\alpha,\beta) := \sum_{n=1}^{N} e\big(\big(\frac{1}{2}n^2+\beta n\big)\,x+n\alpha\big)$ where $\alpha=\frac{a}{2m}$, $\beta=\frac{b}{2m}$ with a,b and m odd, and $\gcd(a,b,m)=1$. We show that there exists a constant $C=C(m)$ such that $|S_N(x;\alpha,\beta)|\le C\sqrt{N}$ for all x. This is joint work with Francesco Cellarosi.

Sudhir Pujahari, University of Warsaw

Distribution of moments of traces of Frobenius in arithmetic progressions

In this talk we will study moments of the trace of Frobenius of elliptic curves if the trace is restricted to a fixed arithmetic progression. We will see an asymptotic formulas for moments for cases for which the prime p goes to infinity for fixed r and cases where the power r goes to infinity with fixed p. As a special case we recover a result of Birch proving Sato-Tate distribution for certain family of elliptic curves. Moreover, we will see that these results follow from similar asymptotic formulas relating sums and moments of Hurwitz class numbers where the sums are restricted to certain arithmetic progressions. This is a joint work with Kathrin Bringmann and Ben Kane.

Anwesh Ray, University of British Columbia

Constructing Galois representations ramified at one prime

A prime p is said to be regular if p does not divide the class number of $\mathbb{Q}(\mu_p)$. Given an integer $n>1$ and a prime p greater than or equal to $4\lfloor n/2\rfloor+1$, Ralph Greenberg constructed a Galois representation $\rho: Gal(\bar{\mathbb{Q}}/\mathbb{Q})\to GL_n(\mathbb{Z}_p)$ with large image with minimal ramification. In fact, ρ is constructed so as to unramified at all primes $\ell\neq p$. We prove that such Galois representations ramified at one prime can be constructed for irregular primes as well. These representations are constructed by lifting suitably chosen residual representations $\bar{\rho}$ with image in the diagonal torus in $GL_n(\mathbb{F}_p)$ for which the associated deformation problem is unobstructed. The results are of interest from the perspective of the Inverse Galois problem.

Manami Roy, Fordham University

Local data for rational elliptic curves with non-trivial torsion

We consider a parameterized family E_T of elliptic curves with the property that they parameterize all elliptic curves $E/\mathbb{Q}$ which contain T in their torsion subgroup, T is one of fourteen possible non-trivial torsion subgroups allowed by Mazur's Torsion Theorem. Using these parameterized families and Tate's Algorithm, we explicitly classify the Néron types, the conductor exponents f_p, and the local Tamagawa numbers c_p of $E/\mathbb{Q}$ at the primes p where it has additive reduction. Consequently, we find all rational elliptic curves with a 2-torsion and 3-torsion point which have global Tamagawa number equal to 1. This is a joint work with Alexander J. Barrios.

Subham Roy, Université de Montréal (student)

Mahler measure of a family of polynomials for arbitrary tori

The (logarithmic) Mahler measure of a polynomial P in n variables is defined as the mean of $\log|P|$ restricted to the standard n-torus ($\mathbb{T}^n = \{(x_1, \ldots, x_n) \in (\mathbb{C}^*)^n : |x_i| = 1, \forall\ 1 \leq i \leq n\}$). The Mahler measure has been related to special values of L-functions, and this has been explained in terms of regulators. Bertin proved several formulas relating the Mahler measure of $K3$-surfaces to the L-function of its transcendental lattice. The generalized Mahler measure is defined as the integral over an arbitrary torus. In this talk, I will discuss few results we obtained involving the relation between the generalized Mahler measure and the standard (logarithmic) Mahler measure of one of Bertin's $K3$-surfaces.

Stelios Sachpazis, Université de Montréal (student)

On multiplicative functions with small averages

There are results of analytic number theory which use information about the prime values of a multiplicative function, in order to extract information about its averages. Examples of such results include Wirsing's theorem and the Landau-Selberg-Delange method. In this talk, we are interested in the opposite direction. In particular, we are going to see that if f is a suitable divisor-bounded multiplicative function with small partial sums, then $f(p) \approx -p^{i\gamma_1} - \ldots - p^{i\gamma_m}$ on average, where the γ_j's are the imaginary parts of the zeros of the Dirichet series corresponding to f. This extends an existing result of Koukoulopoulos and Soundararajan and it is built upon ideas coming from previous work of Koukoulopoulos for the case where $|f| \leq 1$.

Andrew Schultz, Wellesley College

Realizing module structures of square power classes over biquadratic extensions

Recently the Galois module structure for square power classes has been computed in the case where the underlying Galois group is the Klein 4 group. Despite the fact that the modular representation theory allows for an infinite number of non-isomorphic indecomposable types, it turns out that this particular module features at most 9 classes of indecomposable summands. We discuss the realizability of these summand types by drawing ideas from Galois embedding problems and the Brauer group.

Saurabh Singh, Indian Institute of Technology
Delta symbol and its applications
The circle method is a method employed by Hardy, Ramanujan, and Littlewood to solve many asymptotic problems in additive number theory, particularly in deriving an asymptotic formula for the partition function $p(n)$. In this talk we shall
first recall the formula developed by Kloosterman and Duke-Friedlander-Iwaniec and then discuss some applications of the delta symbol.

Eric Stubley, McGill University
Classical forms of low weight in p-adic families of ordinary modular forms
I'll discuss a new strategy for proving theorems of the flavour "a p-adic family of ordinary modular forms contains infinitely many classical weight one forms if and only if it has complex multiplication." The original version of this theorem is due to Ghate–Vatsal, whose proof makes heavy use of the fact that the Galois representation attached to a classical weight one eigenform has finite image. I'll discuss a new proof strategy which avoids using this finite image property, and sketch how this strategy can be applied to new settings such as partial weight one Hilbert modular forms.

John Voight, Dartmouth College
Stickelberger's discriminant theorem for algebras
Stickelberger proved that the discriminant of a number field is congruent to 0 or 1 modulo 4. We generalize this to an arbitrary (not necessarily commutative) ring of finite rank over $\mathbb{Z}$ using techniques from linear algebra. Our proof, which only relies on elementary matrix identities, is new even in the classical case. This is joint work with Asher Auel and Owen Biesel.

Zhining Wei, Ohio State University (student)
Linear Relations of Siegel Poincaré Series and Non-vanishing of the Central Values of Spinor L-functions
In this paper, we will first investigate the linear relations of a one parameter family of Siegel Poincaré series. Then we give the applications to the non-vanishing of Fourier coefficients of Siegel cusp eigenforms and the central values.

Elad Zelingher, Yale University (student)
On regularization of integrals of matrix coefficients associated with spherical models
The Ichino-Ikeda conjecture is a refinement of the Gan-Gross-Prasad conjecture. It roughly states that the Gan-Gross-Prasad period can be expressed as an infinite product of local integrals of matrix coefficients. However, in order to define these local integrals, one must assume that the representations involved are tempered everywhere. In this talk, I will explain how to extend the definition of the local integrals for non-tempered representations for the case of principal series.

Joshua Zelinsky, Hopkins School
Mertens' Theorem and perfect numbers
Given a perfect number N, we are interested in inequalities relating N, $p(N)$, and ω, where p is the smallest prime factor of N, and where $\omega(N)$ is the number of distinct prime factors of N. We will discuss recent results in this direction, and their connection to Mertens' theorem. We will also discuss open question about two functions closely connected to Mertens' theorem.

Peter Zenz, McGill University (student)
Quantum Variance for holomorphic Hecke cusp forms on the vertical geodesic
In this talk we explore a distribution result for holomorphic Hecke cusp forms on the vertical geodesic. More precisely, we show how to evaluate the quantum variance of holomorphic Hecke cusp forms on the vertical geodesic for smooth, compactly supported test functions. The variance is related to an averaged shifted-convolution problem that we evaluate asymptotically. We encounter an off-diagonal term that matches exactly with a certain diagonal term, a feature reminiscent of moments of L-functions.

Luochen Zhao, Johns Hopkins University (student)
Sum expressions for Kubota-Leopoldt p-adic L-functions
An observation due to Delbourgo shows that Kubota-Leopoldt p-adic L-functions can be written as convergent infinite sums, assuming p is odd. We will discuss how these sum expressions hold in general without the assumption, by using integral representations of such p-adic L-functions. Interesting applications, such as a direct proof of the Ferrero-Greenberg formula, will also be presented.