Spectral analysis of the biharmonic operator subject to Neumann boundary conditions on planar dumbbell domains

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A dumbbell domain is the union of two bounded domains joined by a thin channel. We will give an account of recent advances in the spectral analysis of the biharmonic operator Δ^2 subject to Neumann boundary conditions on a dumbbell domain included in $\mathbb{R}^2$. The principal aim is to identify the limit of the eigenvalues and of the eigenprojections as the width of the channel goes to zero. We prove spectral convergence results in the spirit of the articles by J.M. Arrieta et al. for the Neumann Laplacian. In particular we prove that the limiting spectrum is strictly bigger than the sequence of eigenvalues obtained by solving the eigenvalue problem for Δ^2 in the two disjoint domains (corresponding to the dumbbell without the connecting channel). The appearance of these spurious eigenvalues underlines that the thin channel plays an important role even though it has a very small Lebesgue measure. In particular, it is not possible in general to link two disjoint smooth domains via a thin channel without substantially modifying the spectrum. In contrast to what happens with the classical second order case, it turns out that the limiting equation is here distorted by a strange factor.
These results were obtained in collaboration with J.M. Arrieta and P.D. Lamberti.