Spectral bounds for the torsion function, Michiel van den Berg

Let Ω be an open set in a complete, non-compact, m-dimensional Riemannian manifold M with non-negative Ricci curvature, and without boundary, and let v_Ω be the torsion function for Ω. It is shown that v_Ω is bounded if and only if the bottom of the spectrum of the Dirichlet Laplacian acting in $\mathcal{L}^2(\Omega)$, and denoted by $\lambda(\Omega)$, is bounded away from 0. An upper bound for the torsion function is obtained for planar, convex sets in the Euclidean space $M = \mathbb{R}^2$ which is sharp in the limit of elongation. For $M = \mathbb{R}^m,\ m = 2, 3, \dots$ it is shown that the previously obtained bound $\|v_\Omega\|_{\mathcal{L}^\infty(\Omega)}\lambda(\Omega) \ge 1$ is sharp: for any $\epsilon > 0$ we construct an open, bounded and connected set $\Omega_\epsilon \subset \mathbb{R}^m$ such that $\|v_{\Omega_\epsilon}\|_{\mathcal{L}^\infty(\Omega_\epsilon)}\lambda(\Omega_\epsilon) < 1 + \epsilon$.