Free boundary minimal surfaces and large Stekov eigenvalues

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A free boundary minimal surface (FBMS) in the unit ball is a minimal surface $\Omega \subset \mathbb{B} \subset \mathbb{R}^3$ which satisfies one of the two following equivalent conditions:

- Ω meets the boundary $\partial\mathbb{B}$ orthogonally,
- The coordinate functions $x_i : \Omega \to \mathbb{R}$ are Steklov eigenfunctions with eigenvalue $\sigma = 1$.

The study of FBMS is witnessing a renaissance thanks to the fundamental work of Fraser and Schoen, who discovered that isoperimetric optimizers for the first nonzero Steklov eigenvalue $\sigma_1(\Omega)$ of surfaces lead to the existence of FBMS with prescribed topology. For surfaces of genus 0, they proved existence of maximizers for the perimeter-normalized $\sigma_1(\Omega)$ and thereby obtained several new FBMS. In this talk I will show how this link can be used to obtain a sequence of FBMS $\Omega^n \subset \mathbb{B}$ such that $\text{area}(\Omega^n) \xrightarrow{n\to\infty} 4\pi$. This is based on a construction of domains in the sphere $\mathbb{S}^2$ with perimeter-normalized $\sigma_1(\Omega)$ converging to 8π. These domains are obtained by removing small disks from the sphere, in the spirit of homogenization theory. I will also dicuss a conjecture by Fraser and Li, stating that $\sigma_1(\Omega) = 1$ for each FBMS $\Omega \subset \mathbb{B}$. More precisely, I will show that the conjecture is true for some FBMS which are invariant under the action of the symmetry group of some platonic solids.

This talk is based on joint work with Jean Lagacé (UCL).