Coarse nodal count and topological persistence

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Direct generalizations of Courant's celebrated nodal domain theorem are often false. A notable example is the case of linear combinations of eigenfunctions in dimension 2 or more, known as the "Courant-Herrmann conjecture".

It turns out that a bound which is consistent with Courant's theorem and Weyl's law remains true in this case if we ignore nodal domains where the function is small. It also extends in the following sought-after directions: to elliptic operators acting on sections of vector bundles, to higher Betti numbers of nodal domains, and to products of linear combinations of eigenfunctions. In particular, we obtain a coarse version of Bézout's theorem for eigenfunctions.

Our methods combine new results about persistence modules, a notion originating in topological data analysis, and multiscale polynomial approximation in Sobolev spaces.

Joint work with L. Buhovsky, J. Payette, I. Polterovich, L. Polterovich, and V. Stojisavljevic.