OFF-DIAGONAL SPECTRAL CLUSTER ASYMPTOTICS ON ZOLL MANIFOLDS

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In this talk, we consider the asymptotic properties of certain spectral cluster kernels on a smooth, compact Riemannian manifold (M, g) without boundary. Let Δ_g denote the positive Laplace-Beltrami operator on M, which has discrete spectrum $\{\lambda_j^2\}_{j=0}^\infty \subset \mathbb{R}$. We study the Schwartz kernel of the orthogonal projection operator

$$\Pi_{I_\lambda} : L^2(M) \to \bigoplus_{\lambda_j \in I_\lambda} \ker(\Delta_g + \lambda_j^2),$$

where I_λ is an interval centered around $\lambda \in \mathbb{R}$ of a small, fixed length. It is conjectured that on any manifold, $\Pi_{I_\lambda}(x, y)$ has universal asymptotic behavior in a shrinking neighborhood of the diagonal as $\lambda \to \infty$, provided that the interval I_λ is chosen to contain sufficiently many eigenvalues. Such asymptotics then imply that certain statistical properties of monochromatic random waves have universal behavior. The conjecture is known to hold for the round sphere and the flat torus, and also under general geometric assumptions which yield a remainder improvement in the off-diagonal Weyl law. In particular, Canzani and Hanin showed that the conjecture holds near non-self focal points. In this talk, we show that in the opposite case of Zoll manifolds, where all geodesics are periodic with the same period, one can still demonstrate universal asymptotic behavior for an appropriate choice of cluster intervals I_λ.

This talk is based on joint work with Yaiza Canzani and Jeffrey Galkowski.