
Abstract

The main result of this paper is that if a sequence of complex numbers \((a_n)_{n \geq 0}\) satisfies

\[
\sum_{k=0}^{n} \binom{n}{k} a_k = O(n^r) \quad \text{and} \quad \sum_{k=0}^{n} \binom{n}{k} a_k = O(n^r) \quad \text{as} \quad n \to \infty,
\]

for some integer \(r \geq 0\), then \(a_n = 0\) for all \(n > r\). As an application, we deduce a localized form of a theorem of Allan about nilpotent elements in Banach algebras, and this in turn leads to an invariant-subspace theorem. As a further application, we prove a variant of Carleman’s theorem on the unique determination of probability distributions by their moments. The paper concludes with a quantitative form of the main result.