
**Abstract**

Let $(E_0, E_1)$ be a compatible couple of Banach spaces, and let $E_\lambda : 0 \leq \text{Re } \lambda \leq 1$ be the complex interpolation spaces of $E_0$, $E_1$. Let $T$ be a closed linear operator on $E_0 + E_1$, then the restriction $T_\lambda$ of $T$ to each $E_\lambda$ is closed. If we denote by $\tilde{\sigma}(T_\lambda)$ the extended spectrum of $T_\lambda$ in $E_\lambda$, then, under appropriate conditions, it is shown that the map $\lambda \mapsto \tilde{\sigma}(T_\lambda)$ is an analytic multifunction in the strip $\{ \lambda \in \mathbb{C} : 0 < \text{Re } \lambda < 1 \}$. We use these results to give some applications to the spectral theory of semigroups.