Abstract

Let $D$ be a domain of the complex plane containing the origin. The famous great theorem of Émile Picard asserts that if $h$ is holomorphic on $D \setminus \{0\}$, with an essential singularity at 0, then the image under $h$ of any pointed neighbourhood of 0 covers all the complex plane, with at most one exception. Introducing the concept of essential singularity for analytic multifunctions, we extend this theorem to a finite analytic multifunction $K$, of degree $N$, defined on $D \setminus \{0\}$. In this case $\bigcup_{0 < |\lambda| < r} K(\lambda)$ covers all the complex plane, with at most $2N - 1$ exceptions. In particular, this theorem can be used in the case of $N \times N$ matrices whose entries are holomorphic on $D \setminus \{0\}$ with essential singularities at 0. In this case, if their spectra avoid $2N$ points on a pointed neighbourhood of 0, these spectra must be constant.