
Abstract
This paper is a continuation of [1]. We consider the model subspaces $K_\Theta = H^2 \ominus \Theta H^2$ of the Hardy space $H^2$ generated by an inner function $\Theta$ in the upper half plane. Our main object is the class of admissible majorants for $K_\Theta$, denoted by $\text{Adm} \Theta$ and consisting of all functions $\omega$ defined on $\mathbb{R}$ such that there exists an $f \neq 0$, $f \in K_\Theta$ satisfying $|f(x)| \leq \omega(x)$ almost everywhere on $\mathbb{R}$. Firstly, using some simple Hilbert transform techniques, we obtain a general multiplier theorem applicable to any $K_\Theta$ generated by a meromorphic inner function. In contrast with [1], we consider the generating functions $\Theta$ such that the unit vector $\Theta(x)$ winds up fast as $x$ grows from $-\infty$ to $\infty$. In particular, we consider $\Theta = B$ where $B$ is a Blaschke product with "horizontal" zeros, i.e., almost uniformly distributed in a strip parallel to and separated from $\mathbb{R}$. It is shown, among other things, that for any such $B$, any even $\omega$ decreasing on $(0, \infty)$ with a finite logarithmic integral is in $\text{Adm} B$ (unlike the "vertical" case treated in [1]), thus generalizing (with a new proof) a classical result related to $\text{Adm} \exp(i\sigma z)$, $\sigma > 0$. Some oscillating $\omega$'s in $\text{Adm} B$ are also described. Our theme is related to the Beurling–Malliavin multiplier theorem devoted to $\text{Adm} \exp(i\sigma z)$, $\sigma > 0$, and to de Branges’ space $\mathcal{H}(E)$.