
Abstract
Let $G \subset \mathbb{C}^2$ be the open symmetrized bidisc, namely
\[ G = \{(\lambda_1 + \lambda_2, \lambda_1 \lambda_2) : |\lambda_1| < 1, |\lambda_2| < 1\}. \]
We prove that $G$ is not biholomorphic to any convex domain in $\mathbb{C}^2$. By combining this result with earlier work of Agler and Young, we obtain that $G$ is a bounded domain on which the Carathéodory distance and the Kobayashi distance coincide, and yet, not biholomorphic to a convex set.