A SHARP FORM OF THE CRAMÉR–WOLD THEOREM

JUAN ANTONIO CUESTA-ALBERTOS, RICARDO FRAIMAN, AND THOMAS RANSFORD

Abstract. The Cramér–Wold theorem states that a Borel probability measure $P$ on $\mathbb{R}^d$ is uniquely determined by its one-dimensional projections. We prove a sharp form of this result, addressing the problem of how large a subset of these projections is really needed to determine $P$. We also consider extensions of our results to measures on a separable Hilbert space.

Departamento de Matemáticas, Estadística y Computación, Universidad de Cantabria, Spain
E-mail address: cuestaj@unican.es

Departamento de Matemática y Ciencias, Universidad de San Andrés, Argentina
Current address: Centro de Matemática, Universidad de la República, Uruguay
E-mail address: rfraiman@cmat.edu.uy

Département de mathématiques et de statistique, Université Laval, Québec (QC), Canada G1K 7P4
E-mail address: ransford@mat.ulaval.ca

2000 Mathematics Subject Classification. Primary 60E05; Secondary 28C20, 60B11.
Key words and phrases. Probability measures, projections, Cramér-Wold theorem, Hilbert spaces.
First author partially supported by the Spanish Ministerio de Ciencia y Tecnología, grant BFM2002-04430-C02-02.
Second author partially supported by Instituto de Cooperación Iberoamericana, Programa de Cooperación Interuniversitaria AL-E 2003.
Third author partially supported by grants from NSERC and the Canada research chairs program.