ON THE BROWN–SHIELDS CONJECTURE FOR CYCLICITY IN THE DIRICHLET SPACE

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Abstract. Let $D$ be the Dirichlet space, namely the space of holomorphic functions on the unit disk whose derivative is square-integrable. We establish a new sufficient condition for a function $f \in D$ to be cyclic, i.e. for $\{pf : p \text{ a polynomial}\}$ to be dense in $D$. This allows us to prove a special case of the conjecture of Brown and Shields that a function is cyclic in $D$ if it is outer and its zero set (defined appropriately) is of capacity zero.

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