Local maxima of the systole function

Maxime Fortier Bourque
(joint work with Kasra Rafi)

University of Toronto

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Outline

1. Definitions
2. Statement of result
3. Construction
4. Restatement of result

The result

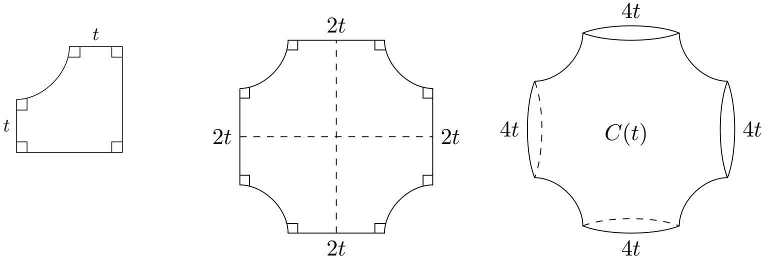
Theorem (0)

There is a linearly growing sequence $(L_n)_{n\geq 1}$ *such that for every* $n \geq 3$ *and certain sufficiently large genera* g*,*

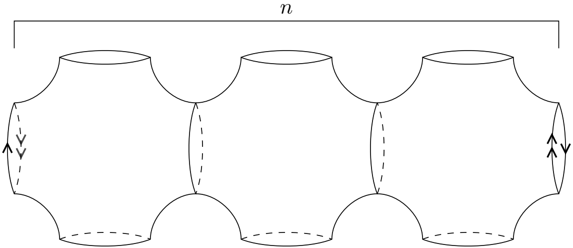
$$\#\{x \in \mathcal{M}_g \mid x \text{ is a local max. of } \mathrm{sys} \text{ and } \mathrm{sys}(x) = L_n\} \geq g^{c(n)g}$$

for some $c(n) > 0$ *independent of* g*.*

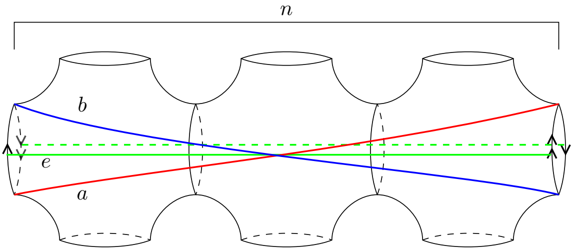
The cross



The ring

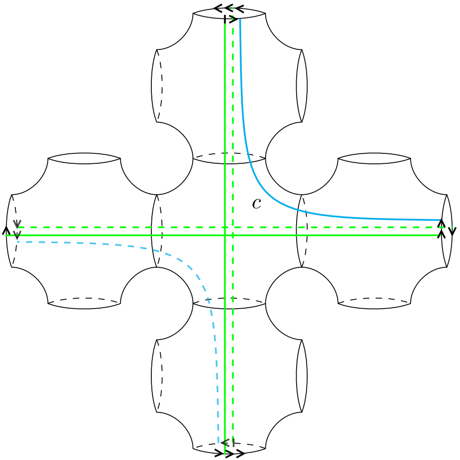


Curves in the ring



There are $2n$ curves of type a and $2n$ curves of type b in the ring. They all have the same length $a(t)$.

Transverse rings

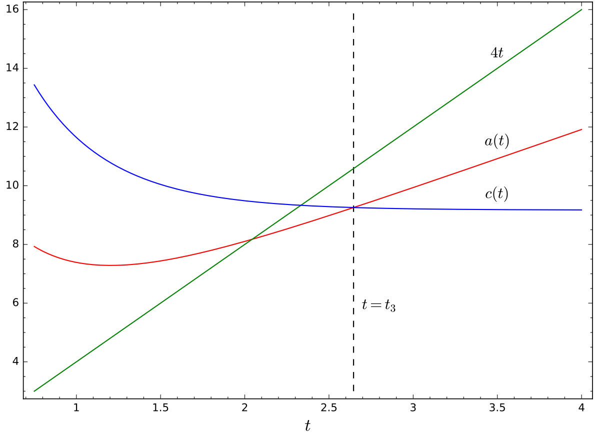


There are 4 curves of type c in the pair of transverse rings.
Their length is denoted $c(t)$.

Choosing t

Lemma

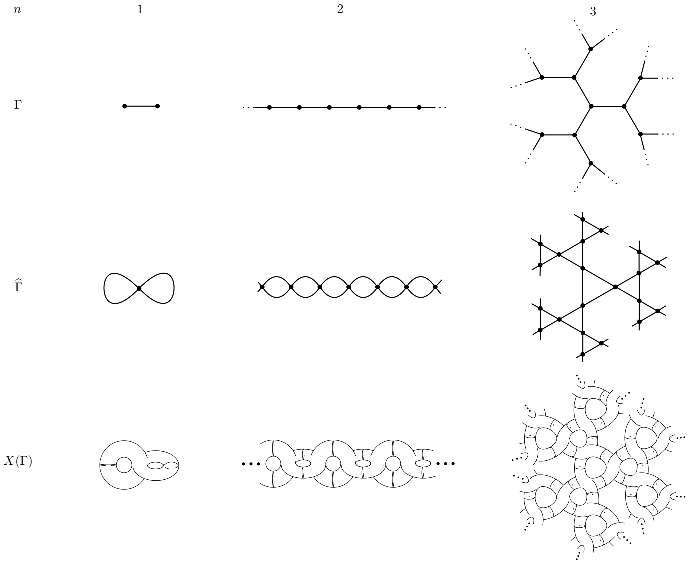
$\forall n \geq 1,\ \exists! t_n > 0,\ a(t_n) = c(t_n).$



How to glue rings together

Let Γ be a connected n-regular graph. We glue crosses/rings with parameter $t = t_n$ together according to the graph Γ as follows.

How to glue rings together



The result, step by step

Theorem (1)

If $\mathrm{girth}(\Gamma) \geq 4n\left(1+\sqrt{2}\right)^n$*, then the systoles in* $X(\Gamma)$ *are the* a*-,* b*- and* c*-curves.*

Theorem (2)

The a*-,* b*- and* c*-curves can detect any deformation of* $X(\Gamma)$*.*

Theorem (3)

Under any deformation of $X(\Gamma)$*, at least one of these curves shrinks.*

$(1)+(2)+(3) \Rightarrow (0)$

Provided Γ has large girth, then $X(\Gamma)$ is a local maximum of sys with value $L_n := a(t_n)$. There are lots of such graphs for $n \geq 3$!

Thank you!